Experimental Gravitation with Cold Atoms

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Cold Atoms and Quantum Engineering
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• Introduction
• Tests of Lorentz Invariance with an atomic fountain clock
• Testing the gravitational inverse square law with trapped atoms
• Variation of fundamental constants?
• ACES/PHARAO
• STE-QUEST
• Conclusion
• General relativity is a classical theory and difficult to reconcile with quantum mechanics and the standard model of particle physics.
• Most unification models predict modifications of gravitational phenomena at some small (generally unknown) level.
• Dark energy and dark matter can be seen as deviations from our known laws of gravitation. A small (but non-zero) value of the cosmological constant ($\Lambda$-CDM model) is incompatible with vacuum energy of quantum field theory.
• Many modified gravitational theories and corresponding cosmological models contain long range scalar fields. Higgs boson is the first known fundamental scalar field (short range).
• Low energy tests of fundamental gravitational physics can provide pieces of the puzzle that are complementary to cosmological observation or high energy physics in accelerators (LHC).
• Precise cold atom metrology allows very sensitive tests of gravitational physics, and may provide first glimpses into physics beyond general relativity and the standard model of particle physics.
Introduction

Quantum Gravity
Unification

String theory
Superstrings
Supersymmetry
Loop Quantum Gravity
M-theory
Brane scenarios
e tc…

Astronomy & Cosmology
(CMB, Planck, EUCLID, …)

Low energy
(EEP tests, inverse square law, …)

High energy
(CERN-LHC, Fermilab, DESY, …)
Test of Lorentz Invariance using the LNE-SYRTE atomic fountain

- search for a dependence of transition frequency as a function of spin orientation
- would indicate an « absolute » reference frame, incompatible with relativity.
- no dependence of the hyperfine transition frequency of $^{133}$Cs au at the $6 \times 10^{-15}$ level.
- best limits on 8 SME parameters (improvements by 11 to 13 orders of magnitude)

The Lorentz violating Standard Model Extension: matter sector

- The matter sector of the SME can be expressed as a perturbation of the standard model Lagrangian, parameterized by 44 parameters (40 at first order in $v/c$) for each known particle ($p^+$, $e^-$, n, in atomic physics).
- Leads to shifts of atomic energy levels as function of the atomic state. In the atom frame:

$$\delta E(m_F, F) = \frac{m_F}{F} \sum_{e^-, p^+, n} \left( \beta_w b^w_3 + \delta_w d^w_3 + \kappa_w g^w_d \right)$$

$$+ \frac{3m_F^2 - F(F+1)}{3F^2 - F(F+1)} \sum_{e^-, p^+, n} \left( \gamma_w c^w_q + \lambda_w g^w_q \right)$$

- $\beta_w, \delta_w, \kappa_w, \gamma_w, \lambda_w$ are specific to the atom and the particular state.
- The tilde coefficients are combinations of SME parameters, to be determined by experiment. They are in general time dependent due to the movement of the atom with respect to a cosmological frame.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$p^+$</th>
<th>$n$</th>
<th>$e^-$</th>
</tr>
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<tbody>
<tr>
<td>$\tilde{b}_X, \tilde{b}_Y$</td>
<td>$10^{-27}$</td>
<td>$10^{-31}$</td>
<td>$10^{-29}$</td>
</tr>
<tr>
<td>$\tilde{b}_Z$</td>
<td>...</td>
<td>...</td>
<td>$10^{-28}$</td>
</tr>
<tr>
<td>$\tilde{b}_T, \tilde{g}_T, \tilde{H}<em>T, \tilde{d}</em>\pm$</td>
<td>...</td>
<td>$10^{-27}$</td>
<td>...</td>
</tr>
<tr>
<td>$\tilde{d}<em>q, \tilde{d}</em>{XY}, \tilde{d}_{YZ}$</td>
<td>...</td>
<td>$10^{-27}$</td>
<td>...</td>
</tr>
<tr>
<td>$\tilde{d}_X, \tilde{d}_Y$</td>
<td>$10^{-25}$</td>
<td>$10^{-29}$</td>
<td>$10^{-22}$</td>
</tr>
<tr>
<td>$\tilde{d}_{XZ}, \tilde{d}_Z$</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\tilde{g}<em>{DX}, \tilde{g}</em>{DY}$</td>
<td>$10^{-25}$</td>
<td>$10^{-29}$</td>
<td>$10^{-22}$</td>
</tr>
<tr>
<td>$\tilde{g}<em>{DZ}, \tilde{g}</em>{JK}$</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\tilde{g}_c$</td>
<td>$10^{-27}$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\tilde{g}_-, \tilde{g}_Q, \tilde{g}_TJ$</td>
<td>$10^{-22(-11)}$</td>
<td>...</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>$\tilde{c}_Q$</td>
<td>$10^{-25}$</td>
<td>$10^{-25}$</td>
<td>$10^{-19}$</td>
</tr>
<tr>
<td>$\tilde{c}_X, \tilde{c}_Y$</td>
<td>$10^{-25}$</td>
<td>$10^{-27}$</td>
<td>$10^{-19}$</td>
</tr>
<tr>
<td>$\tilde{c}<em>Z, \tilde{c}</em>-$</td>
<td>$10^{-21(-8)}$</td>
<td>...</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>
Cs hyperfine Zeeman transitions in the SME

• Using the results of Bluhm et al. [PRD 68, 125008], the perturbation of a
  $|F=3, m_F\rangle \rightarrow |F=4, m_F\rangle$ transition in Cs is:

$$\hbar \delta \omega = B_p \tilde{b}_3^p + D_p \tilde{d}_3^p + G_p \tilde{g}_d^p + C_p \tilde{c}_q^p + B_e \tilde{b}_3^e + D_e \tilde{d}_3^e + G_e \tilde{g}_d^e$$

$$+ Z^{(1)} B + Z^{(2)} B^2$$

\[ \text{SME part} \]

\[ \text{Classical part: } Z^{(1)} B \approx m_F 1400 \text{ Hz} \]

<table>
<thead>
<tr>
<th>$m_F$</th>
<th>$B_p$</th>
<th>$D_p$</th>
<th>$G_p$</th>
<th>$C_p$</th>
<th>$B_e$</th>
<th>$D_e$</th>
<th>$G_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3</td>
<td>-1/6</td>
<td>+1/22 $K_p$</td>
<td>-2/33 $K_p$</td>
<td>+1/14 $K_p$</td>
<td>-3/2</td>
<td>+1/2 $K_e$</td>
<td>-1/2 $K_e$</td>
</tr>
<tr>
<td>-3</td>
<td>+1/6</td>
<td>-1/22 $K_p$</td>
<td>+2/33 $K_p$</td>
<td>+1/14 $K_p$</td>
<td>+3/2</td>
<td>-1/2 $K_e$</td>
<td>+1/2 $K_e$</td>
</tr>
</tbody>
</table>

$K_e \approx 10^{-5}$; $K_p \approx 10^{-2}$ (Schmidt nuclear model).

• Direct measurement limited by first order Zeeman shift (fluctuations of $B$).
• $\Rightarrow$ measure “simultaneously” $\nu_3$, $\nu_{-3}$, and $\nu_0$:

$$\nu_{+3} + \nu_{-3} - 2\nu_0 = \frac{1}{7\hbar} K_p \tilde{c}_q^p - \frac{9}{8} K_z^{(2)} B^2$$

• cancellation of first order Zeeman
• second order Zeeman $\approx -2 \text{ mHz}$
Testing the gravitational inverse square law with trapped atoms: FORCA-G

- look for a modification of the gravitational potential at short range (< 1 µm).
- predicted by unification theories e.g. brane scenarios, compactified dimensions, etc…
- FORCA-G uses cold Rb atoms trapped in a laser standing wave close to a surface (mirror).
- ongoing experiment at LNE-SYRTE.
- complementary to “mechanical” experiments.

$$U(r) = U_N(r)\left(1 + \alpha e^{-r/\lambda}\right)$$

[S. Pelisson, et al., arXiv 1304.6278, (2013), subm. PRA]
FORCA-G: Some experimental results
*(Poster by Pelle & Hilico)*

- First experiments far from the mirror
- Operated on different interferometer config. (Rabi, Ramsey, Accordion, …)
- Good coherence between experimental results and theoretical models
- Detection noise limited performance
- Good progress towards second experiment close to surface!
FORCA-G: Theoretical Model (Poster by Pelisson)

\[ H = H_0 + H_{\text{int}} = H_{\text{WS}} + H_{\text{at}} + H_f + H_{\text{int}} \]

\[ H_{\text{WS}} = \frac{\hbar^2}{2m_a} \frac{d^2}{dz^2} + \frac{U}{2} \left[ 1 - \cos(2k_1z) \right] - m_agz \]

\[ H_{\text{at}} = \hbar\omega_0 |e\rangle\langle e| \]

\[ H_f = \sum_p \int_{0}^{+\infty} dk_z \int d^2k \hbar\omega a_p^\dagger(k, k_z)a_p(k, k_z) \]

\[ H_{\text{int}} = -\mu \cdot E(r) \]

But:

- Perturbation diverges
  ⇒ regularize using finite size of atom
- Include Yukawa term
- Complex scaling approach to calculate lifetimes (> 10^{10} s)
- Fails when including perturbation (regularization?)

\[ E_f/h \approx 8 \text{ kHz} \]
Variation of fundamental constants?

Local Position Invariance (LPI) is a part of the Einstein Equivalence Principle, at the heart of General Relativity. It requires that the outcome of local experiments (e.g., measurement of a fundamental constant) be independent of space-time position.

Unification scenarios (e.g., dilaton models) predict variation of fundamental constants in time and space. More generally one would prefer coupling constants that are determined by some dynamical fundamental field(s) (cf. Kaluza-Klein idea).

Astrophysical observations have claimed different value of $\alpha$ in the past.

Atomic transition frequencies can be related to 3 fundamental dimensionless constants: $\alpha$, $m_q/\Lambda_{\text{QCD}}$, $m_e/\Lambda_{\text{QCD}} (m_e/m_p)$ [Flambaum et al., PRD 2004, PRC 2006].

Measuring the space-time dependence of pairs of transition frequencies allows setting limits on space-time variation of fundamental constants, and thus tests LPI.

J. Guéna et al., PRL 109, 080801, (2012)
LNE-SYRTE Atomic Clock Ensemble

- **Cryogenic Sapphire Oscillator @ 11.932 GHz**
- **Ultrastable reference @ 11.98 GHz**
- **H-masers**
- **NMIs & TAI**
- **GPS**
- **TWSTFT**

**Sr, Hg Optical Clocks & Femtocombs**

- **Cs fountain FO1**
- **Cs mobile fountain FOM**
- **Rb /Cs dual fountain FO2**

**Sr, Hg vs Cs, Rb**
⇒ **LLI test**

**133Cs vs 87Rb hyperfine frequencies**
⇒ **LPI & LLI tests**

**CSO vs maser**
⇒ **LLI test**

**Sr, Hg vs Cs, Rb**
⇒ **LPI test**
Search for a linear drift

Weighted least-squares fit to a line

\[ \frac{d}{dt} \ln \left( \frac{V_{Rb}}{V_{Cs}} \right) = (-1.36 \pm 0.91) \times 10^{-16} \text{ yr}^{-1} \]

Constraint on the drift is improved by \( \sim 7.7 \) wrt PRL 90,150801 (2003)

⇒ With QED calculations: \( J. \text{ Prestage, et al., PRL} \) (1995), \( V. \text{ Dzuba, et al., PRA} \) (1999)

\[ \frac{d}{dt} \ln \left( \frac{g_{Rb}}{g_{Cs}} \alpha^{-0.49} \right) = (-1.36 \pm 0.91) \times 10^{-16} \text{ yr}^{-1} \]

⇒ With QCD calculations: \( T.H. \text{ Dinh, et al., PRA79} \) (2009)

\[ \frac{d}{dt} \ln \left( \alpha^{-0.49} \left( m_q / \Lambda_{QCD} \right)^{-0.021} \right) = (-1.36 \pm 0.91) \times 10^{-16} \text{ yr}^{-1} \]
Search for a modulation with gravitation

- Modulation of the Sun gravitational potential on Earth due to Earth’s orbit ellipticity:

\[ \Delta U(t) \approx -\frac{G M_{\odot}}{a} \epsilon \times \cos \phi(t) \approx 1.6 \times 10^{-10} c^2 \]

Rb/Cs fitted annual modulation:
\[ \Delta \ln(v_{Rb}/v_{Cs}) = (0.18 \pm 1.71) \times 10^{-16} \cos[\Omega_{\odot}(t-t_p)] \]

- Violation of LPI: \( \Delta \nu/\nu = (1+\beta) \Delta U/c^2 \) composition-dependent

\[ \Rightarrow \beta_{Rb} - \beta_{Cs} = (0.1 \pm 1.0) \times 10^{-6} \]

\[ \Rightarrow \begin{cases} \text{With QED calculations:} \\ \text{With QCD} \end{cases} \]

\[ c^2 \Delta \ln( \alpha^{-0.49} \frac{g_{Rb}}{g_{Cs}} ) / \Delta U = (0.1 \pm 1.0) \times 10^{-6} \]

\[ c^2 \Delta \ln( \alpha^{-0.49} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{-0.021} ) / \Delta U = (0.1 \pm 1.0) \times 10^{-6} \]
Global analysis: time variation

\[
\frac{d}{dt} \ln(\nu_1/\nu_2) \approx \Delta k_\alpha \frac{d}{dt} \ln(\alpha) + \Delta k_\mu \frac{d}{dt} \ln(\mu)/dt + \Delta k_q \frac{d}{dt} \ln(m_q/\Lambda_{QCD})
\]

Using most recent calculations of \( k_\alpha \) and \( k_q \) \( \text{Dzuba, Flambaum PRA77,2008; Dinh et al. PRA79, 2009} \)

<table>
<thead>
<tr>
<th>( \nu_1/\nu_2 )</th>
<th>( \Delta k_\alpha )</th>
<th>( \Delta k_\mu )</th>
<th>( \Delta k_q )</th>
<th>( \frac{d}{dt} \ln(\nu_1/\nu_2) \times 10^{-16} \text{ yr}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SYRTE (PRL109, 2012)</td>
</tr>
<tr>
<td>Rb/Cs</td>
<td>-0.49</td>
<td>0</td>
<td>-0.021</td>
<td>-1.36 ± 0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MPQ + SYRTE (PRL92, 2004)</td>
</tr>
<tr>
<td>H(1S-2S)/Cs</td>
<td>-2.83</td>
<td>-1</td>
<td>-0.002</td>
<td>-32± 63</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PTB (PRA 80, 2009)</td>
</tr>
<tr>
<td>Yb+/Cs</td>
<td>-1.83</td>
<td>-1</td>
<td>-0.002</td>
<td>-4.9± 4.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NIST (PRL 98, 2007)</td>
</tr>
<tr>
<td>Hg+/Cs</td>
<td>-5.77</td>
<td>-1</td>
<td>-0.002</td>
<td>3.7± 3.9</td>
</tr>
<tr>
<td>Sr/Cs</td>
<td>-2.77</td>
<td>-1</td>
<td>-0.002</td>
<td>10± 18</td>
</tr>
<tr>
<td>((^{162}\text{Dy}-^{163}\text{Dy})/Cs)</td>
<td>1.7x10^7</td>
<td>-1</td>
<td>-0.002</td>
<td>(-4.0± 4.1)x10^8</td>
</tr>
<tr>
<td>Al+/Hg+</td>
<td>2.95</td>
<td>0</td>
<td>0</td>
<td>-0.53± 0.79</td>
</tr>
</tbody>
</table>

• Least squares fit

\[
\frac{d}{dt} \ln(\alpha) = (-0.25 \pm 0.26) \times 10^{-16} \\
\frac{d}{dt} \ln(m_q/\Lambda_{QCD}) = (71 \pm 44) \times 10^{-16} \\
\frac{d}{dt} \ln(\mu) = (1.5 \pm 3.0) \times 10^{-16}
\]

mainly determined by Al+/Hg+ mainly determined with Rb/Cs .. then determined by Opt/Cs

INDEPENDENT OF COSMOLOGICAL MODELS
Global analysis: variations with gravitation

Variations of frequency ratios with the solar gravitational potential
\[
d/dU \ln(\nu_1/\nu_2) \approx \Delta k_\alpha d/dU \ln(\alpha) + \Delta k_\mu d/dU \ln(\mu) + \Delta k_q d/dU \ln(m_q/\Lambda_{QCD})
\]

<table>
<thead>
<tr>
<th>(\nu_1/\nu_2)</th>
<th>(\Delta k_\alpha)</th>
<th>(\Delta k_\mu)</th>
<th>(\Delta k_q)</th>
<th>(c^2 , d/dU \ln(\nu_1/\nu_2)) ((\times 10^{-6}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb/Cs</td>
<td>-0.49</td>
<td>0</td>
<td>0.021</td>
<td>0.11 ± 1.04 SYRTE (PRL109, 2012)</td>
</tr>
<tr>
<td>H(^f)/Cs</td>
<td>-0.83</td>
<td>0</td>
<td>-0.102</td>
<td>[0.1 ± 1.40] NIST, SYRTE, PTB, INRIM (PRL98, 2007)</td>
</tr>
<tr>
<td>Hg(^+)/Cs</td>
<td>-5.77</td>
<td>-1</td>
<td>-0.002</td>
<td>2.0 ± 3.5 NIST (PRL 98, 2007)</td>
</tr>
<tr>
<td>Sr/Cs</td>
<td>-2.77</td>
<td>-1</td>
<td>-0.002</td>
<td>-12 ± 18 SYRTE, Tokyo, JILA (PRL100, 2008)</td>
</tr>
<tr>
<td>((^{162}\text{Dy-}^{163}\text{Dy})/Cs)</td>
<td>1.7×10(^7)</td>
<td>-1</td>
<td>-0.002</td>
<td>134 ± 104 Berkeley (PRL 2007)</td>
</tr>
</tbody>
</table>

Sr/Cs corrected by factor 2 in value of \(\Delta U/c^2\)

• Least squares fits:

All comparisons:
\[
\begin{align*}
  c^2 \, d/dU \ln(\alpha) &= (0.5 \pm 2.7)\times 10^{-6} \\
  c^2 \, d/dU \ln(\mu) &= (-4 \pm 15)\times 10^{-6} \\
  c^2 \, d/dU \ln(m_q/\Lambda_{QCD}) &= (-5 \pm 28)\times 10^{-6}
\end{align*}
\]

without Rb/Cs, H\(^f\)/Cs:
\[
\begin{align*}
  c^2 \, d/dU \ln(\alpha) &= (1.8 \pm 4.3)\times 10^{-6} \\
  c^2 \, d/dU \ln(\mu) &= \text{not determined} \\
  c^2 \, d/dU \ln(m_q/\Lambda_{QCD}) &= \text{not determined}
\end{align*}
\]

INDEPENDENT OF COSMOLOGICAL MODELS
Atomic Clock Ensemble in Space

ACES

Cs, Rb, Ca, Yb⁺, Sr

Cs, Rb, Sr, Hg H, In⁺, Mg, Ag

Cs, Yb⁺, Yb⁺

Cs, Hg⁺, Al⁺, Sr, Ca, Yb

Cs, Rb

Cs, Rb, Sr⁺, Yb⁺
Cold atom clocks and ground-space comparison links

Launch (Space-X): March 2016 !!

PHARAO

ACES
Primary:
• Demonstrate operation of a next generation space clock and time/frequency links to compare it to ground clocks ($\approx 100x$ improvement on GNSS technology).
• Study laser cooling and manipulation of atoms in microgravity.
• Measurement of the gravitational redshift (test of LPI) to $3\times10^{-6}$ (x25 gain)
• Contribute to the search for a space-time variation of fundamental constants
• Test of Lorentz Invariance (x10 gain)

Secondary:
• Demonstrate relativistic geodesy
• Contribution to TAI (Temps Atomique International)
**STE-QUEST** (ESA preselection 2010, launch ≈ 2022)

**Science Objectives:**
- Earth’s gravitational redshift to $2 \times 10^{-7}$
- Sun and Moon gravitational redshift to $5 \times 10^{-7}$ and $9 \times 10^{-5}$
- Test of the universality of free fall (UFF) using ultra cold ($\approx$BEC) Rb matter waves in differential mode ($^{87}$Rb et $^{85}$Rb) to $2 \times 10^{-15}$
- Tests of Lorentz Invariance
- T/F metrology
- Relativistic geodesy

Period: 16 hrs
Apogee: 51000 km
Perigee: 7100 km
Clock (Cs - PHARAO)
Atom interferometer ($^{85}$Rb-$^{87}$Rb)
Radio and Optical links

Final downselection February 2014
Comparison to tests using macroscopic test masses:

- Different systematics
- In general less sensitive to UFF violation (model dependent !). Eg. in light dilaton scenario (Damour2010) $^{87}$Rb-$^{85}$Rb factor $6 - 56$ less sensitive than Pt-Ti.
- But UFF in quantum context is more general
  - deformation of wave packet, not only C.o.M. motion
  - role of intrinsic spin (anomalous couplings?)
  - superposition state of macroscopic size (few cm)
  - large coherence lengths ($\mu$m) when compared to macroscopic tests ($<10^{-27}$ m)
Cold atoms allow ultra-precise measurements in areas that are intimately related to gravitation (clocks, inertial sensors).

They are therefore particularly well adapted for experimental gravitation, and for exploring physics beyond the standard model and general relativity.

Such experiments allow adding to the increasing body of experimental and observational knowledge coming from other domains like high energy physics, astrophysics and cosmology.

The ensemble of this knowledge constitutes the pieces of a puzzle that will allow us to discover the future laws of physics.

Thank you for your attention!