Measurement of the He-McKellar-Wilkens and Aharonov-Casher phases by atom interferometry

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Summary

Our separated-arm atom interferometer

Measurement of lithium electric polarizability

Topological phases: the Aharonov-Bohm, Aharonov-Casher and He-McKellar Wilkens phases

Simultaneous measurements of the He-McKellar Wilkens and Aharonov-Casher phases

Conclusion
Our atom interferometer
Principle of our lithium atom interferometer

Thermal lithium with velocity near 1000 m/s \( \rightarrow \) de Broglie wavelength \( \approx 55 \) pm
Laser diffraction Bragg regime \( \lambda_L = 671 \) nm \( \rightarrow \) 1st order diffraction angle \( \approx 160 \) microradian

 Atom diffraction by 3 laser standing waves (\( \lambda_L = 671 \) nm)

Observation of interference fringes

\[
I = I_0 [1 + V \cos(\varphi_d)]
\]

\( \varphi_d = \) diffraction phase sensitive to the mirror x-positions

\[
\varphi_d = 2k_L (x_1 + x_3 - 2x_2)
\]
Measurement of a perturbation $U$

Diffraction angle close to 160 microradian
Distance between laser standing waves 0.6 m → the maximum distance between interferometer arms is 100 micrometers.
Each atom goes from the source to the detector by the black and violet paths!

Signal

$$I = I_0[1 + \mathcal{V} \cos(\varphi_d + \varphi_p)]$$

with the perturbation phase $\varphi_p$

$$\varphi_p = \int \frac{U dt}{\hbar}$$
Some experimental details

Atomic beam: supersonic expansion of lithium with a noble gas
- Narrow velocity distribution $\Delta v/v \approx 10\%$
- Tunable mean velocity $v \propto 1/\sqrt{M}$ (M noble gas atomic mass)
- Langmuir-Taylor (hot-wire) detector
- Isotopic selection $^7\text{Li}$ only: natural abundance (92.5%) and laser diffraction
Measurement of lithium electric polarizability
Measurement of lithium electric polarizability $\alpha$

An electric field $E$ on one interferometer arm only $\rightarrow$ perturbation $U = -2\pi \varepsilon_0 \alpha E^2$ $\rightarrow$ induced phase shift $\varphi_{\text{pol}}$

$\varphi_{\text{pol}} = -2\pi \alpha \varepsilon_0 \int \frac{E^2(z)dz}{\hbar \nu}$

$\varphi_{\text{pol}}$ is a dynamical phase shift

The interferometer arms are 30 micrometer wide, 100 micrometers apart

« Septum » : thin foil (6 to 20 micrometers thick) well stretched and placed between the two interferometer arms
Phase shift induced by the electric field

Best measurement of lithium polarizability: $\alpha = 164.2 \pm 1.1$ u.a., in agreement with the best theoretical value $\alpha = 164.1125 \pm 0.0005$ u.a. M. Puchalski (Phys RevA 2012)

Limits on precision: $\varphi_{\text{pol}}$ proportional to $1/v$

accuracy on the mean velocity value
Topological phases: the Aharonov-Bohm, the Aharonov-Casher and the He-McKellar Wilkens phases
The Aharonov-Bohm topological phase (1959)

- it can be detected only by interferometry
- it is independent of the particle velocity
- it changes sign with its direction of propagation

The magnetic field vanishes on the ABF and ACF paths followed by the electron: \( \Rightarrow \) no force but the waves suffer a phase shift \( \varphi \)

\[
\varphi = -\frac{q}{\hbar} \oint_{ACFB} A(r) \cdot dr = -\frac{q}{\hbar} \iint_{ACFB} B(r) \cdot dS
\]

The Aharonov-Bohm phase \( \varphi \) is a topological (or geometric) phase:
- it can be detected only by interferometry
- it is independent of the particle velocity
- it changes sign with its direction of propagation
Experimental study of Aharonov-Bohm phase

Chambers in 1960; Tonomura et al. in 1986

phase shift exactly equal to \( \pi \) (or \( n\pi \)) due to the magnetic flux quantization in the niobium supraconducting ring

FIG. 3. Electron-optical system for hologram formation.

FIG. 2. Toroidal magnet. (a) Scanning electron micrograph; (b) diagram. The toroid is connected to a Nb plate by a tiny bridge for high thermal conductivity.
Aharonov-Casher phase (1984)

Aharonov-Bohm effect with the solenoid replaced by a line of magnetic dipoles

Aharonov-Casher phase: exchange the role of the charge and of the magnetic dipole

Topological phase shift:

$$\varphi_{AC} = -\frac{1}{\hbar c^2} \oint [\mathbf{E}(\mathbf{r}) \times \mathbf{\mu}] \cdot d\mathbf{r}$$
Experimental studies of Aharonov-Casher phase

First test with a neutron interferometer by Cimmino et al. in 1989.

Following experiments with Ramsey or Ramsey-Bordé interferometers with TIF (research group of Ed Hinds in 1992-1993) with Rb (research group of A. Weis in 1995) with Ca (research group of J. Helmcke in 1995)

<table>
<thead>
<tr>
<th>Species</th>
<th>$\mu/\mu_B$</th>
<th>$E_{max}$ MV/m</th>
<th>$\varphi_{AC,max}$ mrad</th>
<th>error bar %</th>
<th>$v$-range m/s</th>
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</thead>
<tbody>
<tr>
<td>Neutron [6]</td>
<td>$1.04 \times 10^{-3}$</td>
<td>30</td>
<td>2.19</td>
<td>24</td>
<td>2680</td>
</tr>
<tr>
<td>TIF [7]</td>
<td>$1.38 \times 10^{-3}$</td>
<td>3</td>
<td>2.22</td>
<td>4</td>
<td>$220 - 340$</td>
</tr>
<tr>
<td>TIF [8]</td>
<td>$1.38 \times 10^{-3}$</td>
<td>2</td>
<td>2.42</td>
<td>2</td>
<td>$188 - 366$</td>
</tr>
<tr>
<td>Rb [9]</td>
<td>$0.333$</td>
<td>0.9</td>
<td>100</td>
<td>1.4</td>
<td>$300 - 650$</td>
</tr>
<tr>
<td>Ca [10]</td>
<td>1.50</td>
<td>1</td>
<td>35</td>
<td>2.2</td>
<td>$643 - 698$</td>
</tr>
</tbody>
</table>
Further generalizations of Aharonov-Bohm topological phase

- Aharonov-Bohm (1959)
- Aharonov-Casher (1984)
- Maxwell duality
  \[ E \leftrightarrow B \]
  exchange electric and magnetic charges
- Dual of Aharonov-Bohm phase
- Dual of Aharonov-Casher phase
  - He-McKellar-Wilkens (1993)
- Exchange electric and magnetic dipoles
  \[ \mu \leftrightarrow d \]
Heuristic explanation of Aharonov-Casher by motional field

\[ \varphi_{AC} = -\frac{1}{\hbar c^2} \oint [\mathbf{E}(\mathbf{r}) \times \mu] \cdot d\mathbf{r} \]

A.G. Klein (1986)

\[ \mathbf{B}_{mot} = -\frac{\mathbf{v} \times \mathbf{E}}{c^2} \]

\[ \varphi_{AC} = \frac{1}{\hbar} \oint \mu \cdot \mathbf{B}_{mot} dt \]

Same argument for the He-McKellar Wilkens phase

\[ \varphi_{HMW} = \frac{1}{\hbar} \oint [\mathbf{B} \times \mathbf{d}] \cdot d\mathbf{r} \]

\[ \mathbf{E}_{mot} = \mathbf{v} \times \mathbf{B} \]

\[ \varphi_{HMW} = \frac{1}{\hbar} \oint \mathbf{d} \cdot \mathbf{E}_{mot} dt \]
Connection between the Aharonov-Bohm phase and the He-McKellar Wilkens phase

Wei et al. (1995), the HMW phase is equal to the algebraic sum of the Aharonov-Bohm phases calculated for the positive and negative charges forming the dipole, interacting with a uniform magnetic field $B$. 
Simultaneous measurements of the He-McKellar Wilkens phase and of the Aharonov-Casher phase
Our goal: detection of the He-McKellar-Wilkens phase

\[ \varphi_{HMW} = \frac{1}{\hbar} \oint [B \times d] \cdot d\mathbf{r} \]
\[ = \frac{4\pi\varepsilon_0\alpha}{\hbar} \oint [B \times E] \cdot d\mathbf{r} \]

Electric field \( E \) (V/m) = 900 V (V in V)
B (T) = 5.6 \times 10^{-4} I (I in A)
\[ \varphi_{HMW} \text{ (rad)} = 1.28 \times 10^{-6} VI \]
Maximum applied voltage $V_{\text{max}} = 800$ V  
Maximum coil current $I_{\text{max}} = 40$ A

$$\varphi_{EB}(V, I) = \varphi_{E+B}(V, I) - \varphi_E(V) - \varphi_B(I)$$

$$= \varphi(V, I) - \varphi(V, 0) - \varphi(0, I) + \varphi(0, 0)$$

Phase drifts $\approx 100$ mrad in 10 minutes and not exactly linear in time

Simultaneous measurements of different field configurations during a 20 second long fringe scan

Uncertainty on $\varphi_{EB}(V, I)$

about 30 mrad after a single fringe scan

about 3 mrad after averaging 100 scans

i.e. 2000 s of data collection
First experiment without optical pumping of lithium

Large stray phase shifts due to several small experimental defects
Zeeman phase shifts (small field difference on the two arms)
Dispersion of polarizability phase shift (geometric defects of the capacitors)

Most defects induce phase shifts which are even functions of the magnetic field
→ cancellation by using results with opposite current values

\[ \varphi_{\text{final}} = \frac{\varphi_{EB}(V, I) - \varphi_{EB}(V, -I)}{2} \]

Measured slope
\[ \varphi_{\text{final}}(V,I) /VI = -(1.68 \pm 0.07) \times 10^{-6} \text{ rad/VA} \]
Predicted slope
\[ \varphi_{\text{HMW}}(V,I) /VI = -(1.28 \pm 0.03) \times 10^{-6} \text{ rad/VA} \]
Optical pumping of the lithium beam in $F=2$, $m_F=+2$ (or $-2$)

Pumping on the D1 line with two laser beams

$m_F=+2$

$m_F=-2$

Test of the pumping efficiency on atom interferometry signals

\[
\begin{array}{c|c|c|c|}
\text{\(v_m\) (m/s)} & m_F & P(2, m_F) \\
\hline
744 \pm 18 & +2 & (96 \pm 6)\% \\
& -2 & (93 \pm 7)\% \\
1062 \pm 20 & +2 & (100 \pm 13)\% \\
& -2 & (95 \pm 11)\% \\
1520 \pm 38 & +2 & (90 \pm 1)\% \\
& -2 & (94 \pm 2)\% \\
\end{array}
\]
Experiment with optical pumping of the lithium beam in the F=2, m_F = +2 or -2 sublevel

As previously, we define:

$$\varphi_{EB}(V, I) = \varphi_{E+B}(V, I) - \varphi_E(V) - \varphi_B(I)$$

$$= \varphi(V, I) - \varphi(V, 0) - \varphi(0, I) + \varphi(0, 0)$$

Now, two measurements for m_F = +2 or -2 → \( \varphi_{EB}(V, I, m_F) \)

\( \varphi_{EB}(V, I, m_F) \) is sensitive to the Aharonov-Casher phase

$$\varphi_{EB}(V, I, m_F) = \varphi_{HMW}(V, I) + \varphi_{AC}(V, I, m_F)$$

$$- \varphi_{AC}(V, I = 0, m_F)$$

\( \varphi_{HMW}(V, I, m_F) \) is independent of m_F while \( \varphi_{AC}(V, I, m_F) \) changes sign with m_F

$$\varphi_{HMW}(V, I) = [\varphi_{EB}(V, I, 2) + \varphi_{EB}(V, I, -2)] / 2$$
Observation of a stray phase shift of unknown origin

Different offsets for opposite voltage values
Already observed in our first measurement of the HMW phase
It does not modify the variation of this phase with the magnetic field
For each series with a given voltage, we subtract the offset.
\( \varphi_{HMW}(V,I) \) as a function of the product \( VI \)

Measured slope \( \varphi_{HMW}(V,I)/VI = -(1.315 \pm 0.071) \times 10^{-6} \text{ rad/VA} \)

Predicted slope \( \varphi_{HMW}(V,I)/VI = -(1.28 \pm 0.03) \times 10^{-6} \text{ rad/VA} \)

Lithium beam mean velocity \( v_m = 1062 \pm 20 \text{ m/s} \)
(carrier gas argon)
Topological character of the He-McKellar-Wilkens phase

3 measurements for $v_m = 744 \text{ m/s}$, $v_m = 1062 \text{ m/s}$ and $v_m = 1520 \text{ m/s}$

variations if $\phi_{HMW} \propto \frac{1}{v^\alpha}$

$\alpha = 1$ : dynamical phase
$\alpha = 2$ : inertial phase
Measurement of the Aharonov-Casher phase

$\varphi_{EB}(V,I,m_F)$ for $m_F = +2$ or -2 is sensitive to the Aharonov-Casher phase

$$\varphi_{EB}(V,I,m_F) = \varphi_{HMW}(V,I) + \varphi_{AC}(V,I,m_F) - \varphi_{AC}(V,I=0,m_F)$$

$\varphi_{AC}(V,I,m_F)$ proportional to $m_F$

$$\varphi_{AC}(V,I,2) - \varphi_{AC}(V,I=0,2) = \left[ \varphi_{EB}(V,I,2) - \varphi_{EB}(V,I,-2) \right] / 2$$

When the coil current $I=0$, the lab field is not vanishing and only the component of $B_{mot}$ along the lab field contributes to $\varphi_{AC}(V,I=0,m_F)$.

separate measurement of $\varphi_{AC}(V,I=0,m_F)$ from the phases in electric field only
Measured values of the Aharonov-Casher phase

Lithium beam in argon: mean velocity $v = 1062$ m/s
Velocity dependence of the Aharonov-Casher phase

\[ \varphi_{AC}/V \times (10^{-4} \text{ rad/V}) \]
Conclusion

• First successful test of the He-McKellar-Wilkens phase 20 years after its theoretical prediction.

• Separated-arm atom interferometers.

• Topological/geometric phases: a quantum curiosity ... but they are very interesting because these phases are non-dispersive.

• A new interferometer under construction for the following experiments

  Test of the electric neutrality of lithium atom with Aharonov-Bohm scalar phase

  Measurement of the electric polarizability of lithium with a topological phase