Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves
## Outline

### Solid state physics scope of the Anderson metal-insulator transition
- The Anderson model of disordered solids
- The Anderson metal-insulator transition

### Experimental realization of an atom-optics system ≡ disordered solid
- The Kicked Rotor with cold atoms
- The quasiperiodic Kicked Rotor

### Experimental demonstration of the Anderson transition
- Direct observation of the crossover
- Characterization of the quantum phase transition
Solid state physics scope
of the Anderson metal-insulator transition
The Anderson model of disordered solids (Anderson, 1958)

\[ V_i \psi_i + W \psi_{i-1} + W \psi_{i+1} = \varepsilon_i \psi_i \]

- \( V_i \) random on-site energy \( \leftrightarrow \) disorder
- \( W \) hopping amplitude

Anderson localization: Electronic states are exponentially localized

- In sharp contrast with the perfect crystal case (Bloch waves)
- Interplay between disorder and interference effects
- No interactions
The Anderson Metal-Insulator transition (Abrahams et al., 1979)

**Theoretical predictions**

- In 1D, wavefunctions always localized ($\forall$ disorder amplitude !)
  - see experiment by J. Billy et al. (Nature, 2008)
- In 3D, disorder induced Metal-Insulator PHASE TRANSITION
  - Numerically observed

**Experimental observation?**

- No interactions
  - Interactions not included in the Anderson model
- No decoherence sources
  - Decoherence break interference effects
- No direct access to the wave-function
  - Rely on modifications of bulk properties (conductivity)
Experimental realization of an atom-optics systems analog to a disordered solid
The Kicked Rotor

\[ H = \frac{p^2}{2} + K \cos x \sum_n \delta(t - n) \]

Classical CHAOTIC DIFFUSION in momentum space

- Looks like a random walk (although perfectly deterministic)
- On average, \( \langle p^2 \rangle \sim Dt \)

\[ \equiv 1D-Anderson \] model (Fishman et al., 1982)

- chaos \( \equiv \) pseudo-random disorder
- momentum \( p \equiv \) site \( i \)
- \( K \equiv \) hopping amplitude

K=10

100 kicks

1000 kicks

10000 kicks
Dynamical localization in momentum space

Compare classical and quantum dynamics of the Kicked Rotor

- At long times, the quantum dynamics freezes

\[ \langle p^2 \rangle |\Psi(p)|^2 \]

(log scale)

Classical chaotic diffusion ⇐ pseudo-random walk

Quantum dynamical localization

Time \( t \) (number of kicks)
How to observe the Anderson transition in 3D?

Dynamical vs. Anderson localization

- Anderson localization: 1D disordered/static/x-space
- Dynamical localization: 1D chaotic/time-periodic/p-space

Simple idea

- Keep the dynamics 1D, but introduce one or several additional temporal dimensions

\[ \hat{H} = \frac{p^2}{2} + k \cos x [1 + \epsilon \cos(\omega_1 t) \cos(\omega_2 t)] \sum_n \delta(\omega_0 t - n) \]

\textbf{Anderson model in higher dimensions (Casati et al., 1989)}

- Prediction for a localized-delocalized Anderson transition when \( K \uparrow \) in a 3 color system
Experimental realization with cold atoms
(F.L. Moore et al., 1994)

\[ \hat{H} = \frac{p^2}{2} + k(t) \cos x \]

- Cesium atoms in a standard MOT
  - Atom-atom interactions negligible
  - thermal distribution=narrow initial distribution (few recoil units)
- Pulsed standing wave
  - \( k(t) \propto I(t) \) laser intensity
  - adjustable effective Planck constant \( \hbar_{\text{eff}} \propto T \)
- Negligible decoherence
  - Controllable spontaneous emission rate \( \propto 1/\text{detuning} \)
- Direct access to the atomic momentum distribution
  - velocity selective Raman technique
Experimental observation of the Anderson transition
From localization to diffusive regime

Numerical simulation

- At criticality: anomalous diffusion $\langle p^2 \rangle \sim t^\gamma$, with $\gamma = 2/3$

$|\psi(p)|^2$

(log scale)

$\langle p^2 \rangle$

Time $t$ (number of kicks)

$K = 9$

diffusive regime

$\langle p^2 \rangle \sim Dt$

$K = K_c$

critical regime

$K = 4$

localized regime

$\langle p^2 \rangle \sim p_{loc}^2$
Experimental observation of the crossover

- **Localized state**: \(\langle p^2 \rangle \sim p_{\text{loc}}^2\)
- **Diffusive regime**: \(\langle p^2 \rangle \sim Dt\)
- **Critical regime**: Anomalous diffusion, \(\langle p^2 \rangle \sim t^{2/3}\)
Experimental final momentum distributions \((t = 150 \text{ kicks})\)

Population

- localized (exponential)
- diffusive (Gaussian)
The Anderson transition

A 2nd order phase transition

- Localization length diverges at $K_c^- : p_{\text{loc}} \sim (K_c - K)^{-\nu}$
- Diffusion constant vanishes at $K_c^+ : D \sim (K - K_c)^s$
- Critical exponents related by: $s = \nu$ in 3D (Wegner's law)

Finite time: experiment $t \leq 200$

- Phase (sharp) transition only observable at the thermodynamic limit: $t \to \infty$
- At finite time, (continous) smooth crossover
- One parameter scaling hypothesis (same as for standard thermodynamic phase transitions): $\langle p^2 \rangle \sim t^{k_1} F \left[ (K - K_c) t^{k_2} \right]$

- Asymtotic behaviours of $F$ ($t \to \infty$)
  \[ \Rightarrow \langle p^2 \rangle \sim t^{2/3} F \left[ (K - K_c) t^{1/3\nu} \right] \]
Critical anomalous diffusion

\[ \langle p^2 \rangle \sim F \left[ (K - K_c) t^{1/3\nu} \right] \Rightarrow \langle p^2 \rangle \sim t^{2/3} \text{ at } K = K_c \]
Finite-time scaling

- Existence of $\xi(K)$ such that: $\frac{\langle p^2 \rangle}{t^{2/3}} = F \left[ \frac{\xi(K)}{t^{1/3}} \right]$ ?
Finite time scaling analysis of numerical results

Scaling function $\mathcal{F}$

$$\Lambda(K, t) = \frac{\langle p^2 \rangle}{t^{2/3}} \sim \mathcal{F} \left[ \frac{\xi(K)}{t^{1/3}} \right]$$

Scaling parameter $\xi$

- $\xi \sim \rho_{\text{loc}}$ for $K < K_c$
- $\xi \sim 1/D$ for $K > K_c$

Critical point: $K_c \approx 6.6$

Critical exponent: $\nu \approx 1.6 \pm 0.05$
Finite time scaling analysis of experimental results

Scaling function $\mathcal{F}$

Scaling parameter $\xi$

First experimental determination of the critical exponent $\nu$

- $1/\xi = \alpha |K - K_c|^{-\nu} + \beta$
- $\beta$ accounts for decoherence effects
- Critical point: $K_c \approx 6.4$
- Critical exponent: $\nu \approx 1.4 \pm 0.3$
- Excellent agreement with numerics (no adjustable parameter)
see also viewpoint in *Physics* **1**, 41 (2008).

- **Experimental realization** of a matter-wave system ≡ 3D Anderson model
- **Direct observation of the crossover** from a localized state to a diffusive regime
- Numerical and experimental proof for the quasiperiodic Kicked Rotor of the **existence of a one parameter scaling function** ⇒ validates the scaling hypothesis
- **Finite-time scaling analysis** ⇒ **first experimental determination of the critical exponent** $\nu$
Critical exponent universal? Same one as for the usual Anderson model? YES

- Critical exponent does not depend on the specific choice of $\hbar$, $\omega_2$ and $\omega_3$
- $\nu \simeq 1.6$ compatible with the numerical value found for the standard 3D-Anderson model ($\nu \simeq 1.58$)

Wave function at critical point. Work in progress

What about two dimensions? Work in progress

Adding additional temporal frequencies $\Rightarrow$ Anderson model in 4, 5... Work also in progress.

Interactions between atoms and dynamical localization?